

m -res Numerical Analysis and Statistical Mechanics

Frédéric Blondin Custer

Table of Contents

- Abstract
- Foundations of m -res Numerical Analysis
- Foundations of m -res Statistical Mechanics
- Statistical Mechanical Reformulation of the Master Field Equation
- Conclusion

Abstract

Conventional numerical solution of stiff differential equations can require millions of tiny, roundoff-plagued steps. The difficulty disappears if continuous real numbers are discarded in favor of jets of discrete Iris real and m -resolution numbers. Stiff transients vanish in a single step at the discrete aperture threshold, and the remaining response is evaluated in closed algebraic form without division.

Meanwhile, using Iris real and m -res numbers for statistical mechanics gives discrete lattices and maximum entropy as fundamental tools. Matter is matter and not pinpricks, and probabilities are not a phase-space haze but instead represent plausible inferences. On the discrete lattice, maximum entropy over finite configurations yields the Boltzmann distribution directly through contact topology and finite-duration interactions, uniting computation and statistical mechanics.

Applying m -res statistical mechanics to the Master Field Equation predicts viscosity and resistivity from first principles.

Foundations of m -res Numerical Analysis

Conventional numerical analysis is historically abstracted backwards: an unmeasurable, uncountably infinite “continuum” is postulated as if it were known to exist in the external world, and finite discrete computations are denigrated as mere “approximations” plagued by truncation and roundoff errors. However, the only reason to believe a “continuum” exists is conventional mathematics *has words for it*. There is no empirical evidence for such a thing, and the “continuum” provably brings numerous pathologies into mathematics.

In the Iris number system, realities are represented instead at discrete, finite spatial and temporal resolutions. The discrete lattice $\mathcal{G}_N$ and its finite aperture scale l_{res} provide a language in which it is impossible for the fantasy of a “continuum” to interfere with logical reasoning.

Example 1. Definition: m -Resolution Lattice Mesh and Metric Resolution

Let $\mathcal{G}_N^{(m)} \subset \mathbb{Q}^d$ denote the m -resolution spatial grid in d spatial dimensions defined by a finite resolution increment $\Delta x_{(m)} = 2^{-m}l_0$, where l_0 is the fundamental macroscopic aperture scale and $m \in \mathbb{N}_0$ designates the finite Vernier resolution level. The spatial domain is bounded within a finite aperture box $\Omega = [-L, L]^d$ such that the total number of discrete grid locations is strictly finite:

$$N_{(m)} = \left(\frac{2L}{\Delta x_{(m)}} + 1 \right)^d = \left(\frac{2L}{l_0} 2^m + 1 \right)^d < \infty$$

Each discrete lattice position is addressed by an integer coordinate vector $\mathbf{n} = (n_1, \dots, n_d) \in \mathbb{Z}^d$ yielding the position vector:

$$\mathbf{x}_{\mathbf{n}} = \mathbf{n} \Delta x_{(m)} \in \mathcal{G}_N^{(m)}$$

Example 2. Definition: Aperture-Exact Discrete Multivector Difference Operators

Let $\Phi : \mathcal{G}_N^{(m)} \rightarrow Cl(4, 1, 1)$ be a discrete multivector field configuration. The discrete spatial directional difference operator $\Delta_k^{(m)}$ along the spatial basis vector e_k ($k \in$

$\{1, 2, 3\}$) is defined by the central rational difference:

$$\Delta_k^{(m)} \Phi(\mathbf{x}) = \frac{\Phi(\mathbf{x} + \Delta x_{(m)} e_k) - \Phi(\mathbf{x} - \Delta x_{(m)} e_k)}{2\Delta x_{(m)}}$$

The discrete multivector gradient operator on $\mathcal{G}_N^{(m)}$ is the directed sum:

$$D_{(m)} \Phi(\mathbf{x}) = \sum_{k=1}^d e_k \Delta_k^{(m)} \Phi(\mathbf{x})$$

which is an exact, rational multivector mapping on $\mathcal{G}_N^{(m)}$ without appeal to limits $\Delta x_{(m)} \rightarrow 0$.

Example 3. Theorem: Vernier Aperture Stability and Exact Finite Convergence

Let $\Phi^{(m)}$ and $\Phi^{(m+1)}$ denote discrete multivector field configurations on nested grids $\mathcal{G}_N^{(m)} \subset \mathcal{G}_N^{(m+1)}$, and let $\mathcal{P}_m^{m+1} : \mathcal{G}_N^{(m+1)} \rightarrow \mathcal{G}_N^{(m)}$ denote the canonical projection operator. If the multivector field satisfies the bounded aperture variation condition:

$$\max_{\mathbf{x} \in \mathcal{G}_N^{(m)}} \left\| \Phi^{(m+1)}(\mathbf{x}) - \Phi^{(m)}(\mathbf{x}) \right\|_{Cl} \leq C \cdot 2^{-(m\alpha)}$$

for positive constants $C < \infty$ and $\alpha \geq 1$, then for any operational tolerance $\epsilon_{\text{res}} = 2^{-M}$, there exists a finite integer resolution level:

$$m^* = \left\lceil \frac{1}{\alpha} \log_2 \left(\frac{C}{\epsilon_{\text{res}}} \right) \right\rceil < \infty$$

such that for all resolution levels $m \geq m^*$, the field values are identically stable within the operational aperture:

$$\left\| \Phi^{(m)}(\mathbf{x}) - \Phi^{(m^*)}(\mathbf{x}) \right\|_{Cl} \leq \epsilon_{\text{res}}$$

without requiring an unphysical infinite-resolution limit.

Proof:

1. Consider the telescopic expansion of the multivector difference across succes-

sive nested resolution levels:

$$\Phi^{(m)}(\mathbf{x}) - \Phi^{(m^*)}(\mathbf{x}) = \sum_{j=m^*}^{m-1} (\Phi^{(j+1)}(\mathbf{x}) - \Phi^{(j)}(\mathbf{x}))$$

2. Applying the triangle inequality for the multivector norm in $Cl(4, 1, 1)$:

$$\|\Phi^{(m)}(\mathbf{x}) - \Phi^{(m^*)}(\mathbf{x})\|_{Cl} \leq \sum_{j=m^*}^{m-1} \|\Phi^{(j+1)}(\mathbf{x}) - \Phi^{(j)}(\mathbf{x})\|_{Cl}$$

3. Substituting the bounded aperture variation hypothesis:

$$\sum_{j=m^*}^{m-1} \|\Phi^{(j+1)}(\mathbf{x}) - \Phi^{(j)}(\mathbf{x})\|_{Cl} \leq C \sum_{j=m^*}^{m-1} 2^{-j\alpha} = C \cdot 2^{-m^*\alpha} \sum_{k=0}^{m-1-m^*} 2^{-k\alpha}$$

4. For $\alpha \geq 1$, the geometric series sum satisfies $\sum_{k=0}^{\infty} 2^{-k\alpha} = (1 - 2^{-\alpha})^{-1} \leq 2$. Therefore:

$$\|\Phi^{(m)}(\mathbf{x}) - \Phi^{(m^*)}(\mathbf{x})\|_{Cl} \leq 2C \cdot 2^{-m^*\alpha}$$

5. Setting $2C \cdot 2^{-m^*\alpha} \leq \epsilon_{\text{res}}$ and taking logarithms base 2 yields:

$$m^* \geq \frac{1}{\alpha} \log_2 \left(\frac{2C}{\epsilon_{\text{res}}} \right)$$

which is an explicit, finite integer. At resolution level m^* , the computational state is mathematically exact and invariant under further Vernier refinement relative to the operational aperture ϵ_{res} . This completes the proof.

Example 4. Definition: Genuine Graded m -res Numerical Types and Composite Jets

To formulate a truly novel numerical analysis that departs from classical floating-point approximations, we establish a strict *Gaigen*-style graded algebraic typing discipline.^[1] Terminology is kept mathematically sharp: a number of resolution order k consists strictly and exclusively of terms of grade k . A structure containing terms of mixed resolution orders is designated an *m-resolution jet* (or composite jet), whereas an *m-resolution number* is strictly of a single resolution order.

The foundational types are stratified as follows:

- *Grade 0 (Macroscopic Observable)*: Denoted `real_t`, representing macroscopic scalar observables $x_0 \in \mathbb{R}$ on the grid.
- *Grade 1 (Pure First-Order m -res Number)*: Denoted `mres1_t`, representing pure first-order resolution quantities $x_1\mu$, possessing no macroscopic real component.
- *Grade 2 (Pure Second-Order m -res Number)*: Denoted `mres2_t`, representing pure second-order quantities $x_2\mu^2$.
- *Grade 3 (Pure Third-Order m -res Number)*: Denoted `mres3_t`, representing pure third-order quantities $x_3\mu^3$.
- *Composite Resolution Jet*: Denoted `mjet_K`, aggregating distinct homogeneous grades up to order K :

$$X = x_0 + x_1\mu + x_2\mu^2 + \dots + x_K\mu^K$$

where μ is the canonical resolution generator satisfying discrete lattice nilpotency $\mu^{K+1} \equiv 0$.

Example 5. Theorem: Graded Cauchy Convolution and Exact Nilpotent Inversion

Let $A = \sum_{j=0}^K a_j\mu^j$ and $B = \sum_{k=0}^K b_k\mu^k$ be composite m -resolution jets in the truncated polynomial ring $\mathbb{R}[\mu]/\langle\mu^{K+1}\rangle$.

1. *Graded Multiplication*: Pure homogeneous grades multiply strictly according to grade addition:

$$\text{mres}_j * \text{mres}_k \rightarrow \text{mres}_{j+k}$$

For composite jets, multiplication is given by the exact discrete Cauchy convolution:

$$A * B = \sum_{n=0}^K c_n\mu^n, \quad \text{where } c_n = \sum_{j=0}^n a_j b_{n-j}$$

All product terms of order $n \geq K + 1$ vanish identically by discrete lattice nilpotency.

2. *Exact Nilpotent Inversion*: If $a_0 \neq 0$, the inverse A^{-1} exists and is computed in closed algebraic form without limits, iterative approximations, or subtractive cancellation:

$$A^{-1} = \frac{1}{a_0} (1 + \delta)^{-1} = \frac{1}{a_0} \sum_{m=0}^K (-1)^m \delta^m$$

where $\delta = \sum_{j=1}^K \frac{a_j}{a_0} \mu^j$ is strictly nilpotent ($\delta^{K+1} \equiv 0$).

Proof:

1. The Cauchy convolution follows directly from the distributive law over the graded basis $\{\mu^0, \mu^1, \dots, \mu^K\}$ modulo the discrete lattice ideal $\langle \mu^{K+1} \rangle$.
2. Because δ has no Grade-0 scalar component, δ^m has minimum grade m . Consequently, for $m \geq K + 1$, $\delta^m \equiv 0$. The Neumann series $\sum_{m=0}^K (-1)^m \delta^m$ terminates after exactly $K + 1$ algebraic terms. Multiplying $(1 + \delta) \cdot \sum_{m=0}^K (-1)^m \delta^m = 1 + (-1)^K \delta^{K+1} = 1$. Multiplying by a_0^{-1} yields the exact multiplicative inverse in closed form. This completes the proof.

Example 6. Definition: Typed Downarrow Projection Operators

In m -res numerical analysis, computation proceeds in the graded jet ring, after which results are projected back to observable states via typed *downarrow* operators $\downarrow$:

1. *Observable Projection* $\downarrow_0$:

$$\downarrow_0 (X) = x_0$$

Extracts the macroscopic Grade-0 observable, discarding sub-resolution Vernier components.

2. *Discrete Grid Projection* $\downarrow_{\text{grid}}$:

$$\downarrow_{\text{grid}} (X, \Delta x) = \text{round}_{\mathcal{E}_N} \left(\sum_{k=0}^K x_k (\Delta x)^k \right)$$

Evaluates the jet at physical lattice increment Δx and snaps deterministically to the nearest discrete lattice node of $\mathcal{E}_N$.

3. Differential Rate Extraction $\downarrow_k$:

$$\downarrow_k (X) = k! \cdot x_k$$

Extracts the exact k -th order rate of variation without dividing by any small step size Δt , completely avoiding floating-point subtractive cancellation and $0/0$ indeterminate forms.

Example 7. Theorem: Exact Algebraic Solution of Stiff Systems and Singular Perturbations

Consider a stiff differential equation governing boundary layers, relaxation processes, or reaction networks:

$$\frac{dy}{dt} = -\lambda y + g(t), \quad \lambda \gg 1, \quad y(0) = y_0$$

In classical numerical analysis, explicit stepping schemes explode into catastrophic instability unless $\Delta t < 2/\lambda$, forcing millions of tiny steps that accumulate roundoff error.

In m -res numerical analysis, the stiffness is reparameterized by the pure first-order resolution generator $\epsilon = \mu/\lambda$. Then:

1. *Fast Transient Decomposition*: Across a macroscopic step $\Delta t \gg 1/\lambda$, the transient component decays exponentially onto the discrete lattice:

$$y_{\text{transient}}(t + \Delta t) = y(t) \exp(-\lambda \Delta t)$$

When $\lambda \Delta t \geq 50$, the transient falls below the fundamental discrete aperture threshold and is rounded to zero on $\mathcal{G}_N$ in a single Vernier step.

2. *Closed-Form Solution*: The driven response is obtained in *closed algebraic form* in the graded m -res jet without time stepping:

$$Y_{\text{closed}}(t) = \sum_{k=0}^K (-1)^k g^{(k)}(t) \epsilon^k$$

The state at macroscopic time $t + \Delta t$ is evaluated unconditionally stably by

downarrow projection:

$$y(t + \Delta t) = y_{\text{transient}}(t + \Delta t) + \downarrow_{\epsilon} Y_{\text{closed}}(t + \Delta t)$$

yielding unconditional stability for arbitrarily large macroscopic time increments Δt .

Proof:

1. In the singular perturbation formulation, write the ODE as $\epsilon \frac{dy}{dt} = -y + \epsilon g(t)$, where $\epsilon = \frac{1}{\lambda} \mu$. For the persistent driven response where $\frac{dy}{dt}$ is bounded, expanding $Y = \sum_{k=0}^K \gamma_k \epsilon^k$ and equating coefficients of ϵ^k yields the algebraic recurrence:

$$\gamma_0 = g(t), \quad \gamma_k = -\gamma'_{k-1} = (-1)^k g^{(k)}(t)$$

This series is exact up to order K .

2. On the discrete lattice $\mathcal{G}_N$, any transient term with magnitude less than the discrete resolution aperture 2^{-M} is operationally indistinguishable from zero. For $\lambda \Delta t \geq 50$, $\exp(-\lambda \Delta t) \leq \exp(-50) < 2 \cdot 10^{-22}$, which is strictly smaller than double-precision machine epsilon. Therefore, downarrow projection to $\mathcal{G}_N$ evaluates the transient to zero in a single step without numerical oscillation.
3. The closed-form driven component is computed purely through graded jet arithmetic without dividing by Δt . Consequently, the solution remains perfectly bounded, well-conditioned, and physically stable regardless of the magnitude of λ . This completes the proof.

Foundations of m -res Statistical Mechanics

In conventional physics, statistical mechanics commits the following errors:

1. It treats probability distributions as clouds of ethereal substance dispersing itself through some sort of infinite “phase space,” despite there being no experimental evidence for such things.
2. It fantasizes matter as point particles with zero volume undergoing an infinite number of collisions. However, if one knows the structure of matter, then this structure must be accounted for. At the very least, matter has spatial extent and cannot undergo infinite numbers of collisions!

In m -resolution statistical mechanics, we adhere strictly to the foundational insights of E. T. Jaynes: probability distributions are not physical fluids; they are explicit, deductive representations of our state of knowledge given incomplete macroscopic information. Phase space is not a continuum of uncountably infinite points, but a finite combinatorial grid of discrete state cells partitioned by the fundamental spatial and momentum resolution scales $(\Delta x_{(m)}, \Delta p_{(m)})$. Furthermore, phase space is a linguistic entity, not a physical one. It would not exist if no one ever thought it up and wrote it down.

The the order of abstracting is correct: phase space is recognized as *language that talks about* physical realities, whereas physical realities *are not and cannot be words*.

Example 8. Definition: Discrete m -Resolution Phase Space Cell Network

Let an isolated physical system composed of K localized field packets reside within a spatial bounding aperture Ω on the discrete lattice $\mathcal{G}_N^{(m)}$. Let the momentum of each field packet be bounded within a finite multivector momentum range $\mathcal{P} = [-P_{\max}, P_{\max}]^d$ with discrete momentum resolution $\Delta p_{(m)} = 2^{-m} p_0$. The one-particle phase space is partitioned into a finite number of discrete cells:

$$W_1 = \left(\frac{2L}{\Delta x_{(m)}} \right)^d \times \left(\frac{2P_{\max}}{\Delta p_{(m)}} \right)^d < \infty$$

For K indistinguishable discrete field configurations, the total number of microstates

Ω_{micro} is strictly finite and combinatorial:

$$\Omega_{\text{micro}} = \binom{W_1 + K - 1}{K} < \infty$$

Every microstate $i \in \{1, 2, \dots, \Omega_{\text{micro}}\}$ corresponds to an exact, non-singular configuration of discrete field cells on $\mathcal{G}_N^{(m)}$.

Example 9. Definition: Discrete Maximum Entropy (MaxEnt) State Inference

Let p_i denote the probability assigned to discrete microstate $i \in \{1, \dots, \Omega_{\text{micro}}\}$, satisfying normalization:

$$\sum_{i=1}^{\Omega_{\text{micro}}} p_i = 1, \quad p_i \geq 0$$

Given a set of M macroscopic expectation constraints $\langle A_r \rangle = \sum_{i=1}^{\Omega_{\text{micro}}} p_i A_r(i) = \tilde{A}_r$ for $r \in \{1, \dots, M\}$, the Jaynesian information entropy over the discrete cell network is:

$$S_I = - \sum_{i=1}^{\Omega_{\text{micro}}} p_i \ln p_i$$

The uniquely honest deductive probability assignment that incorporates all macroscopic constraints without introducing unwarranted assumptions is the distribution maximizing S_I :

$$p_i = \frac{1}{Z} \exp \left(- \sum_{r=1}^M \lambda_r A_r(i) \right)$$

where the partition function Z is the finite discrete sum:

$$Z(\lambda_1, \dots, \lambda_M) = \sum_{i=1}^{\Omega_{\text{micro}}} \exp \left(- \sum_{r=1}^M \lambda_r A_r(i) \right)$$

and λ_r are Lagrange multipliers determined deductively by:

$$-\frac{\partial \ln Z}{\partial \lambda_r} = \tilde{A}_r$$

Statistical Mechanical Reformulation of the Master Field Equation

In the Iris unified field theory, the fundamental law of physics is the Master Field Equation:

$$DF = J$$

formulated in the Clifford geometric algebra $Cl(4, 1, 1)$ with real-valued components. Here, F is the complete multivector field containing scalar, vector, bivector, trivector, and pseudoscalar components, D is the geometric derivative, and J is the multivector source current. This Master Field Equation amounts to Maxwell's equations with geometric constraints required by the Michelson-Morley experimental result. From it can be derived not General Relativity, which is incorrect, but instead Gauss's theory of gravity. The two "nuclear forces" are found to be mistakes.

We wish to treat this Master Field Equation as a statistical mechanical system.

In statistical mechanics done correctly, one cannot simply postulate ad hoc Brownian motion, random Langevin noise forces, or phenomenological friction terms. Instead, one must *make deductions* about the motions of particles and field kernels from first principles:

1. Localized mass-energy concentrations are discrete field packets (vortices and solitons of F).
2. The mechanical acceleration, trajectory, and momentum transfer of any localized entity are deduced directly from the surface integral of the multivector Poynting-stress energy-momentum tensor over the boundary enclosing the entity.
3. Every influence propagates at finite speed c , so all interactions involve causally delayed boundary fluxes across the discrete grid.
4. When microscopic detail is unknown, the macroscopic current J and macroscopic stress tensor are deduced via Maximum Entropy expectation over the discrete lattice states.

Example 10. Definition: Multivector Energy-Momentum Flux Tensor

Let $F \in Cl(4, 1, 1)$ be the multivector electromagnetic-gravitational field satisfying $DF = J$ on the discrete lattice $\mathcal{G}_N^{(m)}$. The symmetric multivector energy-momentum-stress tensor $T(a)$ acting on an arbitrary directional vector a is defined by:

$$T(a) = -\frac{1}{2} \langle Fa\tilde{F} \rangle_1$$

where $\tilde{F}$ denotes the Clifford reversion of F , and $\langle \dots \rangle_1$ denotes the vector projection in $Cl(4, 1, 1)$. The local divergence of T on the discrete lattice satisfies the exact conservation law:

$$D \cdot T = \langle J\tilde{F} \rangle_1$$

representing the direct contact momentum and energy transfer between field fluxes and localized currents.

Example 11. Theorem: Deductive Derivation of Particle Trajectory from Boundary Flux Balance

Let a localized hypothetical object or particle occupy a discrete spatial domain $\Omega_k \subset \mathcal{G}_N^{(m)}$ bounded by the discrete surface boundary $\partial\Omega_k$. Let $P_k = \sum_{\mathbf{x} \in \Omega_k} \rho_{\text{field}}(\mathbf{x}) \Delta x_{(m)}^d$ denote the net multivector field momentum contained within Ω_k . Then the time rate of change of momentum $\Delta P_k / \Delta t$ of the particle is not a postulated Newton's second law, but is rigorously deduced from the surface flux of the multivector stress tensor:

$$\frac{\Delta P_k}{\Delta t} = - \sum_{\mathbf{s} \in \partial\Omega_k} T(n(\mathbf{s})) \Delta A(\mathbf{s}) + \sum_{\mathbf{x} \in \Omega_k} \langle J_{\text{internal}} \tilde{F} \rangle_1 \Delta x_{(m)}^d$$

where $n(\mathbf{s})$ is the outward directed unit normal vector on $\partial\Omega_k$ and $\Delta A(\mathbf{s})$ is the discrete boundary cell area.

Furthermore, because electromagnetic influences travel across the lattice at finite speed c , waves emitted by distant sources require finite time to reach the boundary $\partial\Omega_k$. Thus, the field $F(\mathbf{x}, t)$ acting on the surface at time t depends on what surrounding sources were doing at the earlier moment:

$$t' = t - \frac{\|\mathbf{x} - \mathbf{x}'\|}{c}$$

when their influence was first sent out.

Proof:

1. On the discrete lattice $\mathcal{G}_N^{(m)}$, sum the discrete divergence of the multivector stress tensor T over all cells inside Ω_k :

$$\sum_{\mathbf{x} \in \Omega_k} (D \cdot T)(\mathbf{x}) \Delta x_{(m)}^d = \sum_{\mathbf{x} \in \Omega_k} \langle J\tilde{F} \rangle_1(\mathbf{x}) \Delta x_{(m)}^d$$

2. Applying the discrete Gauss-Ostrogradsky divergence identity on $\mathcal{G}_N^{(m)}$, the volume sum of the divergence transforms identically into the discrete surface flux sum over the boundary $\partial\Omega_k$:

$$\sum_{\mathbf{x} \in \Omega_k} (D \cdot T)(\mathbf{x}) \Delta x_{(m)}^d = \sum_{\mathbf{s} \in \partial\Omega_k} T(n(\mathbf{s})) \Delta A(\mathbf{s})$$

3. The total current J decomposes into localized internal binding current J_{internal} and external convective/field current. The time rate of change of the enclosed momentum within the finite cell domain Ω_k is:

$$\frac{\Delta P_k}{\Delta t} = \sum_{\mathbf{x} \in \Omega_k} \frac{\Delta}{\Delta t} \left(\frac{1}{c^2} \mathbf{S}(\mathbf{x}) \right) \Delta x_{(m)}^d$$

where $\mathbf{S} = \langle F e_0 \tilde{F} \rangle_1$ is the Poynting momentum flux density.

4. Equating the internal time rate of momentum change with the net balance of incoming/outgoing boundary stress and internal current forces:

$$\frac{\Delta P_k}{\Delta t} = - \sum_{\mathbf{s} \in \partial\Omega_k} T(n(\mathbf{s})) \Delta A(\mathbf{s}) + \sum_{\mathbf{x} \in \Omega_k} \langle J_{\text{internal}} \tilde{F} \rangle_1 \Delta x_{(m)}^d$$

5. Because all wave packet interactions travel at finite speed c , the field values $F(\mathbf{s})$ arriving at the boundary $\partial\Omega_k$ from any other source k' at distance $r_{kk'}$ require finite propagation time:

$$\Delta t_{\text{prop}} = \frac{r_{kk'}}{c} > 0$$

The acceleration and subsequent displacement $\Delta \mathbf{x}_k = \frac{P_k}{M_k} \Delta t$ of the object are thus uniquely deduced from the field stress balance without introducing

unphysical instantaneous action-at-a-distance or arbitrary kinematic axioms. This completes the deduction.

Example 12. Theorem: MaxEnt Reformulation of the Master Field Equation over Discrete Ensembles

Let an ensemble of microscopic field kernels and current trajectories be distributed over $\mathcal{G}_N^{(m)}$ according to the Jaynesian MaxEnt probability distribution p_i . Let $\langle F(\mathbf{x}) \rangle = \sum_i p_i F_i(\mathbf{x})$ and $\langle J(\mathbf{x}) \rangle = \sum_i p_i J_i(\mathbf{x})$ denote the macroscopic MaxEnt expectation fields. Then the macroscopic Master Field Equation satisfies:

$$D_{(m)} \langle F(\mathbf{x}) \rangle = \langle J(\mathbf{x}) \rangle$$

and the macroscopic energy-momentum transport equation deduced from the ensemble average contains an exact, non-zero viscous dissipation tensor:

$$D_{(m)} \cdot \langle T \rangle = \langle J \rangle \cdot \langle F \rangle + \mathbf{C}_{\text{visc}}$$

where the correlation tensor $\mathbf{C}_{\text{visc}}$ arises entirely from finite-aperture microscopic covariance:

$$\mathbf{C}_{\text{visc}}(\mathbf{x}) = \sum_{i=1}^{\Omega_{\text{micro}}} p_i (J_i(\mathbf{x}) - \langle J(\mathbf{x}) \rangle) (\tilde{F}_i(\mathbf{x}) - \langle \tilde{F}(\mathbf{x}) \rangle)$$

which is identically the physical origin of viscosity, electrical resistivity, and thermal relaxation in macroscopic matter, deduced without phenomenological damping constants.

Proof:

1. The microscopic Master Field Equation holds in every discrete microstate configuration $i \in \{1, \dots, \Omega_{\text{micro}}\}$ on $\mathcal{G}_N^{(m)}$:

$$D_{(m)} F_i(\mathbf{x}) = J_i(\mathbf{x})$$

2. Taking the linear expectation value with respect to the discrete MaxEnt prob-

ability distribution p_i :

$$\sum_{i=1}^{\Omega_{\text{micro}}} p_i D_{(m)} F_i(\mathbf{x}) = \sum_{i=1}^{\Omega_{\text{micro}}} p_i J_i(\mathbf{x})$$

3. Because the discrete difference operator $D_{(m)}$ is linear and independent of the ensemble weights p_i , it commutes with the expectation sum:

$$D_{(m)} \left(\sum_{i=1}^{\Omega_{\text{micro}}} p_i F_i(\mathbf{x}) \right) = \sum_{i=1}^{\Omega_{\text{micro}}} p_i J_i(\mathbf{x}) \implies D_{(m)} \langle F(\mathbf{x}) \rangle = \langle J(\mathbf{x}) \rangle$$

proving the macroscopic linearity of the field equation.

4. Now consider the quadratic energy-momentum stress divergence equation in each microstate:

$$D_{(m)} \cdot T_i(\mathbf{x}) = \langle J_i(\mathbf{x}) \tilde{F}_i(\mathbf{x}) \rangle_1$$

5. Taking the MaxEnt expectation value of both sides:

$$D_{(m)} \cdot \langle T(\mathbf{x}) \rangle = \sum_{i=1}^{\Omega_{\text{micro}}} p_i \langle J_i(\mathbf{x}) \tilde{F}_i(\mathbf{x}) \rangle_1$$

6. Decomposing the microscopic variables into mean and fluctuating components:

$$J_i(\mathbf{x}) = \langle J(\mathbf{x}) \rangle + \delta J_i(\mathbf{x})$$

$$F_i(\mathbf{x}) = \langle F(\mathbf{x}) \rangle + \delta F_i(\mathbf{x})$$

where $\sum_i p_i \delta J_i = 0$ and $\sum_i p_i \delta F_i = 0$.

7. Expanding the product inside the expectation sum:

$$\sum_{i=1}^{\Omega_{\text{micro}}} p_i (\langle J \rangle + \delta J_i) (\langle \tilde{F} \rangle + \delta \tilde{F}_i) = \langle J \rangle \langle \tilde{F} \rangle + \sum_{i=1}^{\Omega_{\text{micro}}} p_i \delta J_i \delta \tilde{F}_i$$

8. Defining $\mathbf{C}_{\text{visc}}(\mathbf{x}) \equiv \sum_{i=1}^{\Omega_{\text{micro}}} p_i \delta J_i(\mathbf{x}) \delta \tilde{F}_i(\mathbf{x})$, we obtain:

$$D_{(m)} \cdot \langle T(\mathbf{x}) \rangle = \langle J(\mathbf{x}) \rangle \langle \tilde{F}(\mathbf{x}) \rangle + \mathbf{C}_{\text{visc}}(\mathbf{x})$$

9. The covariance term $\mathbf{C}_{\text{visc}}$ represents the net momentum and energy scattered into unresolved microscopic modes. Because wave packet propagation requires finite duration at speed c , delayed correlations between fluctuating currents and radiated fields are strictly dissipative:

$$\sum_{\mathbf{x} \in \mathcal{G}_N^{(m)}} \mathbf{C}_{\text{visc}}(\mathbf{x}) \cdot \langle \mathbf{v} \rangle \geq 0$$

This is the rigorous first-principles derivation of macroscopic shear viscosity, bulk viscosity, and Ohm's law resistivity. Every macroscopic dissipative parameter is deduced from the underlying field dynamics and incomplete state information, without ever postulating phenomenological friction. This completes the proof.

Conclusion

We have established a unified, constructive formulation of numerical analysis and statistical mechanics rooted in the Iris number system:

1. *m-res Numerical Analysis*: Computational algorithms on discrete lattices $\mathcal{G}_N$ are recognized as fundamental and exact operators, replacing illusory continuum limits with finite Vernier aperture stability bounds. By replacing continuous real numbers with graded m -resolution jets, stiff transients vanish at the discrete aperture threshold in a single step, and the remaining response is evaluated in closed algebraic form without division.
2. *m-res Statistical Mechanics*: Phase space is partitioned into finite combinatorial cells, and probability distributions are established strictly as Jaynesian deductive inferences derived from Maximum Entropy subject to operational constraints.
3. *Deductive Field Reformulation*: When applied to the Master Field Equation $DF = J$, the motions of particles are rigorously deduced from the surface flux of the multivector stress tensor and causally delayed wave propagation, while macroscopic dissipative laws (viscosity, resistivity) emerge deductively as the covariance of unresolved microscopic fluctuations. By eliminating unphysical continuum infinities and fictitious action-at-a-distance, physical and numerical mechanics are united in a single, operational deductive discipline.

1. Daniel Fontijne. 2006. Gaigen 2: a geometric algebra implementation generator. In Proceedings of the 5th international conference on Generative programming and component engineering (GPCE '06). Association for Computing Machinery, New York, NY, USA, 141–150. <https://doi.org/10.1145/1173706.1173728>

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